

SEISMIC EXPLORATION

Volume 29

# SEISMIC SIGNATURES AND ANALYSIS OF REFLECTION DATA IN ANISOTROPIC MEDIA

by

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