

DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 53

NONLINEAR ANALYSIS IN SOIL MECHANICS

Theory and Implementation

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