Spectral Analysis and Filter Theory in Applied Geophysics

With 164 Figures and 23 Tables
I Spectral Analysis of Deterministic Processes

1 Fourier Series Representation of Periodic Functions
2 Spectral Representation of Nonperiodic Processes
  2.1 The Fourier Integral
  2.2 Amplitude and Phase Spectra
  2.3 Theorems and Symmetry Properties of the Fourier Transform
  2.4 The Two-Dimensional Fourier Transform
  2.5 The Laplace Transform
  2.6 Determination of the Inverse Fourier Integral by Complex Analysis
  2.7 The Hilbert Transform and the Instantaneous Frequency
3 The Dirac Delta Function and its Fourier Transform
  3.1 Definition of the Delta Function
  3.2 The Fourier Transform of the Delta Function
  3.3 Fourier Transform of a Series of Delta Functions
4 Spectral Analysis of Time-Limited Observations of Infinitely Long Processes
  4.1 The Spectra of Time-Limited Observations
  4.2 Comparison of Weighting Functions
5 Spectral Analysis of Discrete Functions
  5.1 Acquisition of Discrete Data for Continuous Functions
  5.2 Sampling Theorem and Alias Effects
  5.3 Fourier Transform of Discrete Functions (DFT)
5.4 The Fast Fourier Transform ........................................... 72
5.5 The Two-Dimensional Fourier Transform ......................... 78
5.6 Convolution of Time Series ......................................... 79
5.7 Theorems of the Discrete Fourier Transform ..................... 81

6 z-Transform Representation of Time Series 85
6.1 Definition of the z-Transform ...................................... 86
6.2 Convergence of the z-Transform ................................... 87
6.3 Properties of the z-Transform ...................................... 92
6.4 Relationship between the z-Transform and the Discrete Fourier Transform ........................................... 95
6.5 The Inverse z-Transform .............................................. 97

7 Examples of the Use of the Fourier Transform in
Applied Seismics 101
7.1 Frequency-Wavenumber Analysis of Seismic Signals ......... 102
  7.1.1 Frequency-wavenumber representation of seismic signals ........................................... 102
  7.1.2 Examples of frequency-wavenumber analysis .......... 108
7.2 The \( \tau-p \) Transform and its Application in Seismics .... 112
  7.2.1 Principles of the \( \tau-p \) transform ......................... 112
  7.2.2 Numerical calculation of the \( \tau-p \) transform .......... 115
  7.2.3 Decomposition of a wavefield into plane waves ........ 118
  7.2.4 Inverse \( \tau-p \) transform .................................... 120
  7.2.5 Applications of the \( \tau-p \) transform in seismics .... 122
7.3 Migration of Seismic Sections in the
Frequency-Wavenumber Domain ........................................ 126
  7.3.1 Principles of wave-equation migration ................. 127
  7.3.2 Frequency-wavenumber migration ....................... 130
7.4 Calculation of Synthetic Seismograms ............................ 132
7.5 Estimation of the Absorption and Phase Velocity of
Seismic Waves ............................................................. 136
  7.5.1 Determination of the attenuation of seismic waves 136
  7.5.2 Analysis of seismic surface waves ....................... 139

References for Part I 143
II Spectral Analysis of Random Processes

8 Characterization of Random Processes in the Time and Frequency Domains

8.1 Autocovariance and Cross-Covariance Functions

8.1.1 Definitions of the autocovariance and cross-covariance functions

8.1.2 Information provided by the autocovariance and cross-covariance functions

8.1.3 Properties of the autocovariance and cross-covariance functions of random processes

8.1.4 Correlation methods

8.1.5 Summary

8.2 The Power Spectral Density Function

8.2.1 Introduction to the power spectrum

8.2.2 The Wiener-Khinchin transformation theorem

8.2.3 Properties of the power spectral density function

8.2.4 Typical forms of random processes

8.2.5 Examples of the calculation of power spectra

9 Estimation of the Power Spectral Density Function

9.1 Methods for Estimating the Power Spectral Density Function

9.2 Bias and Variance as Quality Indicators of the Estimate of the Spectrum

9.2.1 Bias and variance of the periodogram estimate

9.2.2 Bias and variance of the Blackman-Tukey spectral estimation

9.3 Confidence Limits and Bandwidth of the Blackman-Tukey Spectral Estimate

9.3.1 Determination of the confidence limits

9.3.2 The bandwidth of several spectral windows

9.4 Confidence Limits and Resolution of the Periodogram Estimate

9.5 Practical Determination of the Power Spectra

9.5.1 Algorithms for determining the power spectral density function of discrete sequences

9.5.2 Prewitening of the spectra

9.5.3 Choice of parameter values for the Blackman-Tukey and periodogram estimates
References for Part III 295

IV Fundamentals of Filter Theory 299

13 Filtering from the Viewpoint of System Theory 301
13.1 Types of Filters ........................................ 301
13.2 Impulse-Response and Frequency-Response
       Functions for Characterizing Linear, Time-Invariant
       Filters in the Time and Frequency Domains .......... 302
13.3 Input-Output Relationships of Linear Filters .......... 305
13.4 Inversion of Linear Filters ............................ 308
13.5 Properties of Frequency-Response Functions ........... 308
       13.5.1 Symmetry properties .......................... 308
       13.5.2 The causality condition and its consequences . 309
       13.5.3 Consequences of the stability requirement for
              causal filters .................................. 310
       13.5.4 Spectra of real, stable, causal filters .......... 316
13.6 Special Types of Filters ............................... 317
       13.6.1 Filters without a phase shift .................. 317
       13.6.2 Filters with a linear phase spectrum .......... 318
       13.6.3 Introduction to minimum-phase filters .......... 319
       13.6.4 All-pass filters ............................... 322

14 Filtering in the Frequency Domain 325
14.1 Types of Frequency Filters and their Impulse Response
       Functions ............................................. 327
14.2 Combinations of Linear Filters ........................ 333
       14.2.1 Cascade filters ............................. 333
       14.2.2 Parallel filters ............................. 334
       14.2.3 Examples of filter combinations ............... 334
14.3 Recursive Filters .................................... 338
       14.3.1 First-order recursive filters ................ 339
       14.3.2 Recursive representation of rational filters .. 341

References for Part IV 344
## V  Digital Filtering

### 15 Basics of Digital Filtering
- **15.1 Types of Digital Filters**
- **15.2 The Frequency Response Function of a Digital Filter**
- **15.3 Distribution of Poles and Zeros of Causal and Minimum-Phase Filters**

### 16 Filtering using Simple Mathematical Operations
- **16.1 Filtering using Weighted Averaging**
- **16.2 Compilation of the Filter Effects of Various Mathematical Operations**

### 17 Designing Nonrecursive Digital Filters of Finite Length
- **17.1 Designing Digital Filters by Fourier Series Approximation**
- **17.2 Designing Digital Filters by Sampling the Frequency Response Function**

### 18 Synthesis of Recursive Digital Filters
- **18.1 Design of a Digital Filter by Approximation of an Analog Filter**
- **18.1.1 Approximation of analog filters using the impulse invariance method**
- **18.1.2 The bilinear transform and frequency prewarping**
- **18.2 Design of Recursive Filters by Positioning Poles and Zeros**
- **18.3 Methods for Optimizing the Frequency Response Function for Selected Frequencies**
- **18.4 Summary and Assessment of the Design of Digital Filters**

### References for Part V

## VI  Fundamentals of Optimum Filtering

### 19 Designing Analog and Digital Optimum Filters
- **19.1 The Wiener Optimum Filter**
- **19.2 Solving the Wiener Filter Problem**
- **19.3 Matching Filters to Signals in the Presence of Noise**
19.4 The Effects of Optimum Filters in the Frequency Domain 428
19.5 Design of Digital Optimum Filters 430
19.6 Extension of the Theory of Optimum Filters to Non-stationary Processes 435

20 Application of Optimum Filters to Reflection Seismic Data 439
20.1 Signal Compression using Shaping Filters 441
20.1.1 Shaping filters for white signal sequences 442
20.1.2 Signal contraction when noise is present 444
20.2 Improving the Signal-to-Noise Ratio using Optimum Filters 447
20.2.1 Detection of weak signal arrivals by prediction-error filtering 447
20.2.2 Improving the signal-to-noise ratio using the signal wavelet or its autocovariance function 448
20.3 Summary 453

21 Kalman Filters 455
21.1 State Variables for Describing a System 456
21.2 The Kalman-Bucy Filter for Continuous Functions 462
21.3 Example of the Kalman-Bucy Filter 469
21.4 The Discrete Kalman Filter 474
21.4.1 The Kalman filter algorithm 474
21.4.2 Example of a discrete Kalman filter 481
21.5 Summary 488

References for Part VI 490

VII Fundamentals of Deconvolution and their Application to Reflection Seismic Data 493

22 Mathematical Basis of Deconvolution 495
22.1 Exact Deconvolution 496
22.2 Optimum Deconvolution applying Various Mathematical Criteria 500
22.3 Homomorphic Deconvolution 503
22.3.1 Cepstrum algorithm and inverse filtering 503
22.3.2 Calculation of the complex cepstrum of a time series 504
22.3.3 Cepstrum properties of nonperiodic signals and impulse sequences as a basis for deconvolution 509
22.4 Deconvolution of Nonstationary Time Series 513

23 Deconvolution: Problems and Approaches in Reflection Seismics 515
23.1 Examples of Deterministic Deconvolution 516
   23.1.1 Suppression of seismic ghost reflections 517
   23.1.2 Elimination of water reverberation effects 519
23.2 Deconvolution using Stochastic Models 520
   23.2.1 Estimation of the basic wavelet from the seismogram and determination of signal arrival times 521
   23.2.2 The model assumptions 522
23.3 Predictive Deconvolution 524
23.4 Homomorphic Seismogram Deconvolution 532
23.5 Dynamic Deconvolution 534
23.6 Summary and Outlook 541

References for Part VII 544

VIII Multidimensional and Multichannel Filters 547

24 Multidimensional Filters 549
   24.1 Multidimensional Impulse Response Functions 549
   24.2 Two-Dimensional Filtering of Discrete Fields 551
   24.3 Multidimensional Filtering in Seismics 555
      24.3.1 Geophone arrays as wavenumber filters 557
      24.3.2 The resolution of linear arrays 563
      24.3.3 Velocity and frequency-wavenumber filtering 566

25 Two-Dimensional Filters for Gravity and Magnetic Data 581
   25.1 Smoothing the Measured Data 582
   25.2 Separating Local and Regional Fields in Gravity Data 585
      25.2.1 Separating local and regional field components on the basis of their gravitational effects 586
      25.2.2 Two-dimensional wavenumber filters for gravity data 589
      25.2.3 The second derivative as two-dimensional filter 591
25.2.4 Approximating the second derivative by averaging procedures 595
25.2.5 Separating field components by vertical continuation of the field 596
25.2.6 Summary and discussion 601
25.3 Wavenumber Analysis of Magnetic Data and Estimates of Structure by Downward Continuation of the Field 603

26 Multichannel Filtering of Seismic Data 609
26.1 Basics of Multichannel Filtering 610
  26.1.1 Input-output relationships of multichannel filters 610
  26.1.2 Digital multichannel filters in the time domain 612
26.2 Multichannel Optimum Filters 616
  26.2.1 Fundamentals of multichannel Wiener optimum filters 617
  26.2.2 Comparison of the most commonly used multichannel optimum filters 625
  26.2.3 Designing optimum stacking filters using stochastic models 635
26.3 Summary 638

References for Part VIII 640

Author Index 645

Subject Index 651