

DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 38

SOIL PLASTICITY

Theory and Implementation

W.F. CHEN

School of Civil Engineering, Purdue University, West Lafayette, IN 47907, U.S.A.

and

G.Y. BALADI

U.S. Army Engineering Waterways Experimental Station, Vicksburg, Miss., U.S.A.



ELSEVIER

Amsterdam — Oxford — New York — Tokyo 1985



CONTENTS

Preface	v
Notation	xii
<i>Chapter 1 THE CONTINUUM THEORY OF SOIL MECHANICS</i>	1
1.1 Introduction	1
1.2 Notations	2
1.3 Stresses in three dimensions	3
1.3.1 Definitions and notations	3
1.3.2 Cauchy's formulas, index notation, and summation convention	6
1.3.3 Principal axes of stresses	8
1.3.4 Deviatoric stress	10
1.4 Strains in three dimensions	11
1.4.1 Definitions and notations	11
1.4.2 Deviatoric strain	12
1.4.3 Octahedral strains and principal shear strains	12
1.4.4 Relationship between engineering strain and natural strain	13
1.5 Equations of solid mechanics	15
1.5.1 Equations of equilibrium (or motion)	15
1.5.2 Geometry (compatibility) conditions	16
1.5.3 Constitutive relations	17
1.5.4 Mathematical analysis	18
1.6 Constitutive modeling of soils and rocks	20
1.6.1 Soil and rock as a continuum	20
1.6.2 General approaches	21
References	21
<i>Chapter 2 ELASTIC-PLASTIC CONSTITUTIVE MODELING OF SOILS</i>	23
2.1 Elastic and plastic theories applied to design in soil	23
2.1.1 Idealizations	23
2.1.2 Elasticity and modeling	24
2.1.3 Plasticity and modeling	25
2.2 Soil plasticity – a brief historical sketch	26
2.2.1 Metal plasticity	26
2.2.2 Soil plasticity	27
2.2.3 The literature on the state-of-the-art	28
2.3 Failure criteria	29
2.3.1 Failure surfaces in three dimensions	29
2.3.2 Strength models	30
2.3.3 Mohr-Coulomb criterion in three dimensions	32
2.3.4 Advantages and limitations	33
2.4 Cauchy elasticity and modeling	33
2.4.1 Linear elasticity	33

2.4.2 Nonlinear elasticity	35
2.4.3 Cauchy elastic material	36
2.4.4 Cauchy elastic models	36
2.4.5 Advantages and limitations	36
2.5 Hyperelasticity and modeling	37
2.5.1 Hyperelastic material	37
2.5.2 Hyperelastic models	38
2.5.3 Advantages and limitations	39
2.6 Hypoelasticity and modeling	39
2.6.1 Hypoelastic material	39
2.6.2 Hypoelastic models.	41
2.6.3 Advantages and limitations	42
2.7 Deformation plasticity and modeling	43
2.7.1 Deformation theory of plasticity	43
2.7.2 Deformational plastic models	43
2.7.3 Variable moduli models.	45
2.7.4 Advantages and limitations	46
2.8 Incremental plasticity and modeling	47
2.8.1 Flow theory of work-hardening plasticity	47
2.8.2 Flow theory of perfect plasticity	48
2.8.3 Advantages and limitations of perfectly plastic models	50
2.8.4 Limit analysis of perfect plasticity	50
2.9 Isotropic hardening plasticity and modeling	51
2.9.1 Hardening (softening) rules	51
2.9.2 Concept of end cap	53
2.9.3 Cambridge models (concept of critical state).	54
2.9.4 Generalized cap models.	55
2.9.5 Advantages and limitations	56
2.10 Kinematic hardening plasticity and modeling	56
2.10.1 Concept of nested yield surfaces	56
2.10.2 An illustrate example.	57
2.10.3 Mixed-hardening nested yield surface model.	58
2.11 Mixed hardening plasticity and modeling	59
2.11.1 Concept of bounding surface	59
2.11.2 General remarks	60
References	61
 <i>Chapter 3 NONLINEAR ELASTIC-PERFECTLY PLASTIC MODELS</i>	
3.1 Introduction	65
3.2 Basic concept of plasticity	65
3.3 General description of elastic-perfectly plastic constitutive relations	66
3.3.1 Elastic strain increment tensor.	66
3.3.2 Plastic strain increment tensor	68
3.3.3 Total strain increment tensor	71
3.4 Prandtl-Reuss material.	72

3.5 Drucker-Prager material	79
3.6 Procedure for fitting elastic-perfectly plastic models to a given set of material properties	84
3.7 Numerical implementation of elastic-perfectly plastic models	85
3.7.1 Numerical algorithm	85
3.7.2 Derivation of the stiffness matrix	88
3.8 User's Guide for computer program MODEL	90
3.8.1 Glossary	90
3.8.2 Guide to data input	91
3.8.3 Brief flow chart of the program MODEL	92
3.8.4 Listing of the program MODEL	92
3.9 User's Guide for the triaxial driver	95
3.9.1 Listing of the triaxial driver	95
3.9.2 Numerical example	101
3.10 Mechanical behavior of soil	101
3.11 Ability of ideal plastic models to simulate soil behavior	108
3.12 Capped yield surfaces	110
References	111

Chapter 4 NONLINEAR ELASTIC WORK-HARDENING PLASTIC CAP MODELS

4.1 General description of the cap model	113
4.2 Derivation of the general incremental strain-stress relations	115
4.2.1 The elastic strain increment tensor	115
4.2.2 The plastic strain increment tensor	116
4.2.3 Total strain increment tensor	118
4.3 Derivation of the stiffness matrix for the cap model	118
4.4 Demonstration of selected forms of the cap model	120
4.4.1 Behavior of the selected cap model under triaxial test conditions; isotropic consolidation phase	123
4.4.2 Behavior of the selected cap model under triaxial test conditions; shear phase	125
4.5 Procedure for fitting elastic work-hardening plastic models to a given set of material properties	128
4.6 Numerical implementation of the cap model	129
4.7 User's guide for the model subroutine	137
4.7.1 Model subroutine	137
4.7.2 Listing of a computer program for the cap model	138
4.7.3 Listing of the triaxial driver	143
4.7.4 Listing of computing initial position of the cap	149
4.8 Numerical example	150
References	155

Chapter 5 NUMERICAL EXAMPLES

5.1 Nonlinear analysis	157
----------------------------------	-----

5.1.1 Finite-element analysis	157
5.1.2 Numerical implementation of models	157
5.1.3 NFAP – an overview	158
5.1.4 Flow chart for NFAP	158
5.1.5 General remarks	160
5.2 Active and passive earth pressures	161
5.2.1 Strain path and history dependent behavior	162
5.2.2 Model predictions	164
5.3 Settlement and collapse calculations of footings	167
5.3.1 Von Mises material	167
5.3.2 Drucker-Prager material	169
5.4 Large deformation analysis of slopes	170
5.4.1 Analyses of slopes prior to seismic loadings	171
5.4.2 Analyses of slopes during seismic loadings	172
5.4.3 Analyses of slopes after slidings	177
References	180

Chapter 6 ADVANCED CAP MODELS

6.1 Introduction	183
6.2 Generalized total stress model for isotropic materials	183
6.2.1 The total stress model	183
6.2.2 Comparisons of laboratory test data with model behavior	185
6.3 Strain-softening model	188
6.3.1 Elastic behavior	189
6.3.2 Plastic behavior	189
6.3.3 Correlation with test results	191
6.4 Elastic-viscoplastic model	192
6.4.1 Fundamental basis of elastic-viscoplastic constitutive model	193
6.4.2 Correlation of test data with model behavior	196
6.5 An elastic-plastic transverse-isotropic model	200
6.5.1 Fundamental definitions of “pseudo stress invariants”	200
6.5.2 Plastic behavior	200
6.5.3 The elastic behavior	202
6.5.4 Flow rule and proportionality factor	204
6.5.5 Basic features of the model	204
6.5.6 Comparisons of laboratory test data with model behavior	210
6.6 Effective-stress model	214
6.6.1 Effective-stress concept	214
6.6.2 The treatment of a two-phase system	216
6.6.3 Application	217
6.6.3.1 Comparison of laboratory data with model prediction	220
References	225
Author index	227
Subject index	229