

*Developments in Geotechnical Engineering 30*

# MECHANICS OF PARTICULATE MATERIALS THE PRINCIPLES

by

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