

DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 61

# FINITE ELEMENT TECHNIQUES IN GROUNDWATER FLOW STUDIES

WITH APPLICATIONS IN HYDRAULIC  
AND GEOTECHNICAL ENGINEERING

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